

2008年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2008

学科試験 問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数 学

MATHEMATICS

注意 ☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Fill in the blanks with the correct numbers or expressions.

1) Solve the equation $3x^3 - 10x^2 + 10x - 4 = 0$.

$x =$

2) Solve the simultaneous inequality $\begin{cases} 3x^2 - 8x - 3 \geq 0 \\ 2x^2 - 11x + 9 < 0 \end{cases}$.

3) Solve the equation $\log_2(x-1) - \log_4(x+3) = \frac{1}{2}$.

$x =$

4) Solve the inequality $2\sin^2 x > 3\cos x$ ($0 \leq x < 2\pi$).

5) Let α and β be the roots of the equation $2x^2 - 5x + 1 = 0$, where $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $x^2 + ax + b = 0$. Find the values of the constants a and b .

$a =$, $b =$

6) Let $\alpha = 2$, $\beta = \sqrt{3} + i$ and $\gamma = 1 + i$ ($i^2 = -1$). Find the absolute value r and the argument θ of $\frac{\alpha + \beta}{\gamma}$ ($-\pi < \theta \leq \pi$).

$r =$, $\theta =$

7) Calculate $\sum_{k=1}^n (k+1)(k+2)$.

8) Suppose $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{3}$ and $|\vec{a} - 2\vec{b}| = 2$. Find the angle θ between \vec{a} and \vec{b} ($0 \leq \theta \leq \pi$).

9) Calculate $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$.

10) Suppose $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{kx}\right)^x = \sqrt{e}$. Find the value of the constant k .

11) Find the derivative $\frac{dy}{dx}$ of $y = x^{\sin x}$.

12) Calculate $\int_0^1 x^2 e^x dx$.

2 Let $B = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$, $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and A be a matrix satisfying $A^2 = O$,
 $AB = O$.

1) Calculate $(I+A)(I-A)$.

2) Calculate $(I+A)^{-1}(I+2A)^{-1}B$.

3) When $A = \begin{pmatrix} 1 & x \\ y & z \end{pmatrix}$, find x, y, z .

3 Let C be the curve defined by $y = \sqrt{ax+b}$ where a, b are constants. Suppose the point $A(0,2)$ is on the curve C and the tangent to the curve C at the point A intersects the x -axis at the point $B(-8,0)$.

1) Find a and b .

2) Let l_1, l_2 be the tangents to the curve C which pass through the point $P(-1,2)$. Find the equations of l_1, l_2 .

3) Evaluate the area of the figure enclosed by C, l_1, l_2 .